

Endogenous Growth, R&D and Labour Variety

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1 INTRODUCTION

Among the great variety of endogenous growth models, only a few have been constructed to address the question of how economic growth is related to conditions in the labour market and how growth is related to unemployment. We think that these questions are important and that the little attention paid to them represents a serious gap in the existing literature.

The purpose of this paper is to go some way towards ...lling this gap by developing a model which can be use to study the roles of labour market policies and institutions in determining growth.

1.1 Outline of the Main Endogenous Growth Models

The importance of growth is due to the effect of it on the standard of living. Small differences in the growth rate when accumulated for long periods of time have stronger consequences for standard of living than the short term fluctuations which have worried economists for a long time. Thus, economists should be interested in why some countries grow faster than others; how a

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country with low growth rates can be transformed in a country with high growth rates; and what determines the long-run growth rate.

The growth literature started with the model of Harrod-Domar and the Solow or neo-classical model. But due to the impossibility of providing a satisfactory answer to those questions by these two models a new literature, both theoretical and empirical, emerged in the 1980's. This is the literature on endogenous growth.

The main difference between the neo-classical model and endogenous growth models is that the former explains growth by appealing to exogenous technological progress, while the latter explains growth in terms of structural parameters of the economy, including those parameters describing policy. In other words, models of endogenous growth determine growth within the model.

Another difference is that, in the neo-classical model, there is necessarily convergence of per-capita incomes among identically structured countries. That is, poor or less developed countries will grow faster than rich or developed countries until the income level is equal in both. However, in the endogenous growth models, there exists the possibility that rich countries will always remain rich and poor countries will always remain poor.

The endogenous growth literature began with the article of Romer (1986) and Lucas (1988). Romer's model is a model with learning by doing and knowledge spillovers. This model is based on Arrow (1962), and assumes that diminishing returns to capital can be eliminated due to the link between the state of knowledge and the amount of investment. This means that if a firm increases its physical capital it is able to learn how to produce more efficiently, because an increase in the capital stock increases the stock of knowledge. There is another important assumption in this model, namely that knowledge is a public good, so that new knowledge is available instantly for the whole economy. These two assumptions change a neo-classical growth model into an endogenous growth model.

Lucas' model is based on Romer (1986) and on the Uzawa (1965) model. It is a model with two sectors. The first sector is a final good sector where physical and human capital are combined in order to produce the final good which can be consumed or changed into physical capital. In the second sector, the production and accumulation of capital take place from physical and human capital. The model assumes that the ways of producing physical capital and human capital are different. The most important assumption is non-diminishing returns in the production of human capital, because the

sustained positive growth rate is driven by this sector.

Another model of endogenous growth, the AK model, was introduced by Rebelo (1991). In this model the production function shows constant returns to scale and constant returns to capital. The growth rate depends on the saving rate and on the productivity of the technology. The saving rate is determined by the patience of society and the intertemporal elasticity of substitution. Higher patience and higher elasticity mean larger savings and growth rate.

In all these models there is no technological progress, and growth can “go on indefinitely because the returns to investment in a broad class of capital goods -which include human capital- do not necessarily diminish as economies develop” (Barro and Sala-i-Martin (1995), p. 12)

However, although in all those models we find long term growth “the mere accumulation of capital -even a broad concept that includes human capital- cannot sustain growth in the long run because this accumulation must eventually encounter a significant decline in the rate of return” (ibid., p.212). Then, if we want to develop a model of endogenous growth where diminishing returns to capital can exist we have to introduce technological progress. The models which include technological progress are characterised by the existence of Research and Development firms and imperfect competitive markets. These models began with Romer (1987, 1990) with subsequent important contributions by Grossman and Helpman (1991) and Aghion and Howitt (1992). They are called models of product innovation. In this kind of models, there is technological progress as a result of R&D activity. The R&D activity is encouraged by the existence of monopoly power for the designs. In these models, governments have important consequences on the long-run growth because they can affect positively or negatively the growth rate through taxation, provision of infrastructures, protection of intellectual property rights, financial markets, etc. We can classify the models of product innovation into two categories, depending on the aim of the R&D firms.

The first type of models is that in which the new good is used for new functions, that is, there is an increase in the variety of goods for consumption and/or production. In these models there is long run growth because the R&D firms always wish to find new products because of the existence of positive profits as a result of the imperfect competitive markets. This type of model will be used later.

The second type of models is that in which the aim of the R&D firms is to increase the quality of the existing goods. They are based on the idea of

quality ladders because it is supposed that every good can be improved an unlimited number of times. As in the prior case, R&D firms are interested in developing better quality goods in order to get positive profits in the imperfect competitive market.

1.2 Growth and Labour Market: A Quick Review of the Literature

We can see that all the previous models of growth do not examine any relation between labour market and growth. Only a few economists like Pissarides (1990, ch. 2), Bean and Pissarides (1993) and Aghion and Howitt (1994) developed models to cover that gap in the literature. The little attention paid to this question may be “consistent with the seminal theoretical work of Phelps (1968) which implies that the natural rate of unemployment is independent of the rate of productivity growth” (Aghion and Howitt (1994), p. 477).

Pissarides (1990, ch. 2) analyses how a change in the productivity of growth rate affect the equilibrium unemployment. His model implies that an increase in the growth of productivity produce a reduction in the natural rate of unemployment, because there is an increase in the rate of return to the creation of vacancies and, therefore, unemployment is reduced.

Bean and Pissarides (1993) developed a model with job matching and where firms’ technology exhibits decreasing returns to their own capital but the aggregate technology is linear in aggregate capital. Firstly, they obtain that a reduction in the hiring costs produce an increase in the number of vacancies which implies a reduction in the unemployment. The effect of this lower unemployment is higher saving and then higher growth. Secondly, they look at the effect of an increase in the workers’ bargaining power. They find that the consequence of this is a higher unemployment, although the effect on growth is ambiguous. Finally, they study the effect of an increase in the marginal propensity to consume. The result in this case is surprising because saving and growth can increase, which is in opposition with the classical theory where the increase in the propensity to consume produce a fall in savings and growth, as a consequence of the mark up prevailing in the consumption goods market.

Aghion and Howitt (1994) developed a model with technological progress and job matching. They say that we can find two effects of growth on unem-

ployment: the capitalisation effect and the creative destruction effect. The capitalisation effect is the effect that we described in Pissarides (1990, ch. 2), that is, a higher growth rate produce an increase in the rate of return from creating a new job, so new firms enter in the market, and more vacancies are created causing reduction in the unemployment. The creative destruction effect is based on the idea that when productivity growth occurs, low productivity jobs are replaced by new high productivity ones, thereby increasing unemployment. This effect is a reallocation effect which works in opposite direction to the capitalisation effect. Aghion and Howitt say that the capitalisation effect dominates at high growth rates, while the reallocation effect does it when the growth rates are low.

1.3 Organisation of the Paper

The rest of the paper is organised as follows. In section 2 we establish the structure of the model that we focus on. In section 3 we derive the balanced growth equilibrium and examine the properties of this equilibrium. Section 4 contains some policy implications. Concluding remarks are presented in section 5.

2 THE MODEL

We use a standard overlapping generations model with constant population. Agents are heterogeneous representing differentiated types of workers. Agents live for two periods. In the first period, when young, they work in order to earn a wage, and in the second period, when old, they consume their savings. As in Blackburn and Hung (1993) there are three sectors of production: a final goods sector where a single consumption good is produced; an intermediate goods sector where different kind of intermediate goods are produced; and a Research and Development sector where new intermediate goods are invented and designed. All agents and firms are price takers except the intermediate goods firms which act in monopolistic competition. The final consumption good is the numeraire.

2.1 Households

Households maximise the lifetime utility function:

$$U^t(j) = \ln \mathbb{E} C_t^t(j)^\alpha + \mu \ln [1 - l_t(j)] + (1 + \frac{1}{2})^{-1} \ln \mathbb{E} C_{t+1}^t(j)^\alpha; \quad (1)$$

where $C_t^t(j)$ is consumption of generation t of type j worker when young and $C_{t+1}^t(j)$ is consumption of generation t of type j worker when old. $l_t(j)$ is the quantity of j th type labour supplied by generation t (so $1 - l_t(j)$ is the amount of leisure). $\frac{1}{2}$ is the discount factor or rate of time preference ($\frac{1}{2} > 0$); μ is a parameter which represents the taste for leisure ($\mu > 0$); and \mathbb{E}_t is the expectation operator.

The maximisation of (1) is subject to the constraints:

$$C_t^t(j) + a_{t+1}(j) = w_t(j)l_t(j) \quad (2)$$

$$C_{t+1}^t(j) = a_{t+1}(j)E_t(1 + r_{t+1}) \quad (3)$$

where $w_t(j)$ is the wage of type j labour and r_{t+1} is the real interest rate.

Solving the problem we get the optimality condition for consumption:

$$\frac{E_t C_{t+1}^t(j)}{C_t^t(j)} = \frac{E_t(1 + r_{t+1})}{1 + \frac{1}{2}}; \quad (4)$$

We assume that each type of workers belongs to a union which recognises a degree of monopoly power. Therefore, the union takes into account the effect of labour supply on the wage when solving workers decision problems. This effect can be expressed in terms of the elasticity of demand for labour as:

$$\frac{1}{1 + \frac{1}{2}} = \frac{\frac{\partial w(j)}{\partial l(j)} l(j)}{w(j)}; \quad (5)$$

Then the relation between consumption when young and the supply of labour (or demand of leisure) is:

$$C_t^t(j) = \frac{1}{\mu} w_t(j) [1 - l_t(j)]; \quad (6)$$

2.2 Final Goods Sector

The production function follows the specification of Dixit and Stiglitz (1977), Ethier (1982), Romer (1987, 1990) and is a modification of the extension of the constant return to scale Cobb-Douglas technology used by Blackburn and Hung (1993). This production function is:

$$Y_t = \left(\sum_{j=1}^J l_t^F(j)^{\frac{1}{1-\sigma}} \right)^{1-\sigma} Z_t^{\frac{1}{1-\sigma}} \int_0^{M_t} [x_t(i)]^{\sigma} di \quad (7)$$

where Y_t is final output, $l_t^F(j)$ is the quantity of the j th type of labour input, j is the number of different types of labour, $x_t(i)$ is the i th type of intermediate good and $i \in [0; M_t]$, M_t being the last created intermediate good. The elasticity of substitution between any two kinds of labour is $\frac{1}{1-\sigma}$, and the elasticity of substitution between any two goods is $\frac{1}{1-\sigma}$.

In this model, technological progress is determined by an expansion of M_t , the number of differentiated types of intermediate goods, which raises the efficiency of other inputs (labour). Observe that an increase in J (the number of different labour types) and a decrease in σ (the degree of differentiation among labour types) are measures of labour variety. Just as an increase in product variety improves the efficiency of labour, so too does an increase in labour variety improve the efficiency of existing intermediate goods.

The firm has to hire labour from households and buy intermediate goods from the producers of that sector. It has to pay a wage $w_t(j)$ for the j th type of labour and a price $p_t(i)$ for the i th type of intermediate good. The price of the final good is the numeraire. Then the profit maximisation problem is:

$$\max_{x_t(i); l_t^F(j)} \pi_t^F = \left(\sum_{j=1}^J l_t^F(j)^{\frac{1}{1-\sigma}} \right)^{1-\sigma} Z_t^{\frac{1}{1-\sigma}} \int_0^{M_t} [x_t(i)]^{\sigma} di - \sum_{j=1}^J w_t(j) l_t^F(j) - \int_0^{M_t} p_t(i) x_t(i) di \quad (8)$$

Solving this problem we obtain the expression for the labour demand and the demand for intermediate goods:

$$l_t^F = \frac{(1-\sigma) Y_t [w_t(j)]^{-\frac{1}{1-\sigma}}}{W_t} \quad (9)$$

where $W_t = \prod_{j=1}^J [w_t(j)]^{\frac{1}{1+\alpha_j}}$;

$$x_t = \frac{Y_t [p_t(i)]^{\frac{1}{1+\alpha_i}}}{P_t} \quad (10)$$

where $P_t = \int_0^{M_t} [p_t(i)]^{\frac{1}{1+\alpha_i}} di$:

2.3 Intermediate Goods Sector

As in the final good sector, the production function for intermediate goods is an extension of the Cobb-Douglas technology. The production function for the h th type of intermediate good is:

$$z_t(h) = \left(\prod_{j=1}^J l_t^j(j; h)^{\frac{1}{1+\alpha_j}} \right)^{\frac{1+\alpha_z}{1+\alpha_z}} \int_0^{M_t} x_t^i(i; h) di; \quad (11)$$

where $l_t^j(j; h)$ and $x_t^i(i; h)$ are the quantities of the j th type of labour and the amount of the i th type of intermediate good respectively, used in producing the h th type of intermediate good.

The firm minimises the following cost function:

$$\min_{x_t^i, l_t^j} \sum_{j=1}^J w_t(j) l_t^j(j; h) + \int_0^{M_t} p_t(i) x_t^i(i; h) di \quad q_t(h) \quad (12)$$

subject to (11). $q_t(h)$ is the fee that the firm must pay to the R&D firm in order to use the design for good h , because the patent is held by the R&D firm.

The Lagrangian is:

$$H = \sum_{j=1}^J w_t(j) l_t^j(j; h) + \int_0^{M_t} p_t(i) x_t^i(i; h) di - q_t(h) - \lambda \left(\left(\prod_{j=1}^J l_t^j(j; h)^{\frac{1}{1+\alpha_j}} \right)^{\frac{1+\alpha_z}{1+\alpha_z}} \int_0^{M_t} x_t^i(i; h) di - z_t(h) \right); \quad (13)$$

where λ is the lagrangian multiplier. From the first order conditions we obtain the quantity demanded of labour and the quantity demanded of intermediate goods

$$l_t^j(j; h) = \frac{(1 - \alpha) \lambda z_t(h) [w_t(j)]^{\frac{1}{1-\alpha}}}{W_t} \quad (14)$$

where $W_t = \sum_{j=1}^P [w_t(j)]^{\frac{1}{1-\alpha}}$;

$$x_t^i(i; h) = \frac{\alpha \lambda z_t(h) [p_t(i)]^{\frac{1}{\alpha-1}}}{P_t}; \quad (15)$$

where $P_t = \int_0^M [p_t(i)]^{\frac{\alpha}{\alpha-1}} di$;

We also obtain the marginal cost, represented by the lagrangian multiplier:

$$\lambda = \frac{1}{1 - \alpha} \frac{W_t}{P_t} \frac{1}{\alpha} \quad (16)$$

The intermediate goods market is characterised by monopolistic competition. Each intermediate goods firm differentiates its product from the other firms. This differentiation permits the firm to charge a price equal to the profit maximisation monopoly rental rate $p_t(h)$. Therefore, each intermediate goods firm has to solve the following maximisation problem:

$$\begin{aligned} \max_{p_t(h)} \pi_t^h &= p_t(h) z_t(h) - \lambda z_t(h) - q_t(h) \\ \text{s.t: } z_t(h) &= x_t^F(h) + \int_0^M x_t^i(i; h) di \end{aligned} \quad (17)$$

where the left hand side of the restriction is the total production of the intermediate good h , and the right hand side is the sum of the intermediate good h demands by the final goods sector and the intermediate goods sector. The expression for $z_t(h)$ using (15) and (10) is:

$$z_t(h) = \frac{\alpha p_t(h)^{\frac{1}{\alpha-1}} (Y_t + \alpha Z_t)}{P_t} \quad (18)$$

where $Z_t = \int_0^{M_t} z_t(h) dh$.

Then, the maximisation problem (17) can be written as:

$$\begin{aligned} \max_{p_t(h)} \quad & \int_0^{M_t} [p_t(h) - \frac{1}{\theta}] z_t(h) - q_t(h) \\ \text{s.t. } \quad & z_t(h) = \frac{[p_t(h)]^{\frac{1}{1-\theta}} (Y_t + \frac{1}{\theta} Z_t)}{P_t} \end{aligned} \quad (19)$$

Solving this problem, we find, as expected, that the price follows the constant mark-up rule

$$p_t(h) = \frac{1}{\theta} \quad (20)$$

Competition among firms will drive profits to zero, in which case we also have:

$$q_t(h) = \frac{(Y_t + \frac{1}{\theta} Z_t) \frac{1}{\theta} (1 - \frac{1}{\theta})^{\frac{1}{1-\theta}}}{P_t} \quad (21)$$

2.4 Research and Development Sector

In the Research and Development sector there are N_t risk neutral firms. These firms invent designs for new intermediate goods. Research is conducted using labour. We call $I_t^R(j; k)$ the amount of j th type labour in the k th R&D firm, and let M_t represent the current quantity of generally available knowledge.

The probability of innovation is defined by:

$$\frac{\theta}{M_t} \prod_{j=1}^{\theta} I_t^R(j; k)^{\frac{1}{\theta}} = \frac{\theta}{M_t} \quad (22)$$

where $\theta \in [0, 1]$ and $\theta'(\theta) > 0$ and $\theta''(\theta) < 0$.

Firms pay workers if the project is successful. The state of each firm is private information, only the firm itself can directly observe the outcome of its research project. This raises a problem since a firm would always like to claim to be in a bad state. We think of the solution to this problem in terms of an incentive compatible contract which requires that firm makes a fixed payment, $f_{t+1}(j)$, if the project succeeds and which is enforced through

costly verification. Given this, the expected discount value of profits of a firm is

$$V_t^R(k) = \sum_{j=1}^J M_t \sum_{k=1}^K l_t^R(j; k) \left[\sum_{j=1}^J L^R(j; k) f_{t+1}(j) \right] \frac{1}{1 + r_{t+1}} \quad (23)$$

where $L^R(j; k)$ is the number of type j workers and

$$Q_t(k) = \sum_{s=1}^S (1 + r_{t+s})^{-1} q_{t+s}(k) \quad (24)$$

Let $\phi(A; J; \gamma)$ be the per project cost to workers of monitoring the firm (i. e. the cost of verifying whether the project has succeeded or failed). We assume that $\phi_A(\phi) > 0$ (for some $A > 0$); $\phi_J(\phi) > 0$ and $\phi_\gamma(\phi) < 0$. The last two properties capture the idea that a greater variety of labour makes it more difficult and costly for each type of worker to evaluate the success of a project.

Given the above, we may define the cost of each project to each worker of type j as:

$$\hat{A}(j; k) = \frac{\phi(A; J; \gamma)}{L(j; k) + \gamma \sum_{j=1}^J L(j; k)} \quad (25)$$

where $\gamma(J; \gamma) \in (0; 1)$, $\gamma_J < 0$ and $\gamma_\gamma > 0$. The denominator of expression (25) capture the idea that the cost can be shared by workers of the same type and possibly by workers of different types according to γ . As above, we assume that greater labour variety makes cost sharing among different types of workers more difficult.

Now we can define the incentive or participation constraint for workers to be willing to work in R&D in the following way:

$$\begin{aligned} & \sum_{j=1}^J M_t \sum_{k=1}^K l_t^R(j; k) \left[\sum_{j=1}^J L^R(j; k) f_{t+1}(j) \right] \hat{A}(j; k) (1 + r_{t+1}) \\ & = w_t(j) \bar{l}_t^R(j; k) (1 + r_{t+1}) \end{aligned} \quad (26)$$

where $\bar{l}_t^R(j; k)$ is the number of hours of a type j worker in the k th firm of the R&D sector.

The constraint means that the expected return from working in R&D must be equal to the safe return from working in manufacturing. If the expected return from working in the R&D sector is lower than the return from working in the manufacturing sector, workers have an incentive to leave the R&D sector in order to work in the manufacturing sector.

If we multiply both sides of the constraint by $\prod_{j=1}^P L(j; k)$ and define employment as

$$l_t^R = L(j; k) \bar{l}_t^R(j; k); \quad (27)$$

we get:

$$\frac{\sum_{j=1}^P L(j; k) f_{t+1}(j)}{1 + r_{t+1}} \leq \sum_{j=1}^P L(j; k) \bar{A}(j; k) = \sum_{j=1}^P w_t(j) l_t^R(j; k) \quad (28)$$

and the maximisation problem is now:

$$\max_{l_t^R} \pi_t^R(k) = \sum_{j=1}^P Q_t(k) \leq \sum_{j=1}^P w_t(j) l_t^R(j; k) \leq \bar{A}(k) \quad (29)$$

where $\bar{A}(k) = \prod_{j=1}^P L(j; k) \bar{A}(j; k)$.

The R&D firm equates expected marginal revenue to marginal cost. This is obtained from the first order condition. Furthermore, the existence of free entry drives equilibrium profits to zero. These two conditions are respectively:

$$\sum_{j=1}^P M_t \left(\sum_{j=1}^P l_t^R(j; k)^{\alpha_j} \right)^{\frac{1}{1-\alpha_j}} l_t^R(j; k)^{\alpha_j - 1} Q_t(k) = w_t(j) \quad (30)$$

$$\sum_{j=1}^P Q_t(k) = \sum_{j=1}^P w_t(j) l_t^R(j; k) + \bar{A}(k); \quad (31)$$

From (31) we see that a fixed cost is introduced into research and development. This cost reflects the essential activity of monitoring in the presence of asymmetric information. The importance of this cost will become apparent later.

With (30) and (31) we can obtain the amount of j th type labour demanded by the k th R&D firm:

$$l_t^R(j; k) = \frac{\frac{h}{\gamma} \frac{(j)w_t(j)}{(j)M_t} + \tilde{A}(k) (w_t(j))^{-\frac{1}{\gamma}}}{W_t}; \quad (32)$$

3 BALANCED GROWTH EQUILIBRIUM

In balanced growth equilibrium, all agents and firms are optimising, all markets are clearing and all non-stationary variables are growing at the same rate. We call this growth rate g .

Given the symmetry in the model, all firms in each sector have the same prices and quantities so that we may omit the index on variables. As in Blackburn and Hung (1993), we will define the following terms: $w^a = \frac{w_t}{M_t}$ is the constant growth adjusted wage; $n = \frac{N_t}{M_t}$ is the constant ratio of the potential flow of new designs to the stock of existing designs; $L^I(j) = M_t l_t^I(j)$ is the constant total amount of the j th type of labour employed in the intermediate sector; $L^R(j) = N_t l_t^R(j)$ is the total constant amount of the j th type labour employed in the research and development sector;

$e = M_t \sum_{j=1}^J l_t^R(j) = M_t J l_t^R = \frac{M_t J L^R}{N_t}$ is the constant total number of efficiency units of input in the R&D sector. Using the last expression, we can see that $\sum_{j=1}^J M_t l_t^R(j) = e$. We also define: $Z_t = M_t z_t$;

$P_t = M_t (p_t)^{\frac{1}{1-\gamma}}$; and $W_t = J (w_t)^{\frac{1}{1-\gamma}}$.

With the previous expressions and with (15) and (20) we can get:

$$Z_t = \frac{\gamma^2 Y_t}{(1 - \gamma) M_t^{\frac{1}{1-\gamma}}}; \quad (33)$$

Introducing (33) in (14) we obtain the amount of labour demanded by

the intermediate sector:

$$L^I(j) = \frac{\pi^2 Y_t}{J W^\alpha M_t^2 (1 + \pi)} \quad (34)$$

Then, introducing (33) in (21) we find the value of :

$$q_t = \pi \frac{(1 + \pi) Y_t}{M_t} + \frac{\pi^2 Y_t}{(1 + \pi) M_t} \quad (35)$$

From equations (9) and (32) we obtain the amount of labour demanded by the final sector and R&D sector respectively:

$$L^F(j) = \frac{(1 + \pi) Y_t}{J M_t W^\alpha} \quad (36)$$

$$L^R(j) = \frac{N_t \frac{h}{\pi(e)} W^\alpha + \tilde{A}(k)}{M_t J W^\alpha} = \frac{ne}{J^\frac{1}{\alpha}} \quad (37)$$

Now with (34), (35), (36) and (37) we obtain the total amount demanded of jth type labour:

$$L^D(j) = L^F(j) + L^I(j) + L^R(j) = \frac{q_t}{\pi J W^\alpha} + \frac{ne}{J^\frac{1}{\alpha}} \quad (38)$$

Due to the independence of the probability of successful innovation, the increase in new designs is:

$$M_{t+1} - M_t = \pi(e) N_t \quad (39)$$

Since steady state growth occurs at the rate:

$$g = \frac{M_{t+1} - M_t}{M_t} \quad (40)$$

then,

$$g = \pi(e) n \quad (41)$$

From (4), and assuming that the interest rate is constant we get

$$\frac{C_{t+1}}{C_t} = 1 + g = \frac{1 + r}{1 + \frac{1}{2}} \quad (42)$$

which can be approximated by

$$g \approx \frac{1}{4} r i^{-1/2} \quad (43)$$

From the definition of Q_t , we see that

$$Q_t = \frac{q_t}{r}; \quad (44)$$

and introducing (44) into (30) we obtain:

$$q_t = \frac{r W^{\frac{1}{\alpha}} J^{\frac{1-\alpha}{\alpha}}}{s^{\frac{1}{\alpha}}(e)} \quad (45)$$

Then, introducing (45) into (38), the total amount of labour demanded is:

$$L^D(j) = \frac{1}{J^{\frac{1}{\alpha}}} \frac{r}{s^{\frac{1}{\alpha}}(e)^{\frac{1}{\alpha}}} + n e^{\frac{1}{\alpha}}; \quad (46)$$

Now we will determine labour supply. For the j th type labour, the supply is equal to demand. Hence,

$$L(j)l_t(j) = \frac{1}{J^{\frac{1}{\alpha}}} \frac{r}{s^{\frac{1}{\alpha}}(e)^{\frac{1}{\alpha}}} + n e^{\frac{1}{\alpha}} \quad (47)$$

where $L(j)$ is the total amount of households supplying the j th type of labour, and $l_t(j)$ is the number of hours of the j th type of labour supplied by a household. If we define the total number of households as $\bar{L} = \sum_{j=1}^P L(j) = J L$ (assuming $L(j) = L$), then the total labour supply is:

$$\bar{L}l = J^{\frac{1}{\alpha}} \frac{r}{s^{\frac{1}{\alpha}}(e)^{\frac{1}{\alpha}}} + n e^{\frac{1}{\alpha}}; \quad (48)$$

With (43) and (48) we obtain

$$\frac{r}{s^{\frac{1}{\alpha}}(e)^{\frac{1}{\alpha}}} J^{\frac{1}{\alpha}} \bar{L} i^{-1/2} + n e^{\frac{1}{\alpha}} = s^{\frac{1}{\alpha}}(e) n + \frac{1}{2}; \quad (49)$$

This equation shows a relation among l , e and n . But, in order to find the values of those three variables, we need another two equations.

We use (30) and (31) to obtain the second equation. From (30) we know that

$$e_s(e)Q(k)J^{\frac{1}{1-\alpha}} = w^\alpha; \quad (50)$$

and from (31), after some changes, we get

$$e_s(e)Q(k) = J^{\frac{1}{1-\alpha}} w^\alpha e + \tilde{A}(k) \quad (51)$$

Now, dividing (50) by (51), we find the expression

$$\frac{e_s(e) \frac{1}{e} e_s(e)}{e_s(e)} w^\alpha = J^{\frac{1}{1-\alpha}} \tilde{A}(k); \quad (52)$$

which relates e and w^α . If we find the value of w^α , we will have an expression which depends only on e . To do that, first we introduce the value of \pm , given by (16), in (20) and we obtain the price. Therefore, the price is:

$$p = \frac{J^{\frac{1}{1-\alpha}} w^\alpha M_t^{\frac{\alpha}{1-\alpha}}}{(1 - \alpha) \frac{\alpha}{1-\alpha}}; \quad (53)$$

Now, we introduce the value of the price in (9), finding the amount demanded of x^F :

$$x^F = \frac{(1 - \alpha) \frac{\alpha}{1-\alpha} Y}{J^{\frac{1}{1-\alpha}} w^\alpha}; \quad (54)$$

Then, with the value of x^F , I^F and the expression (1) we get the value of w^α . Therefore,

$$w^\alpha = J^{\frac{1}{1-\alpha}} \bar{\omega} \quad (55)$$

where $\bar{\omega} = \frac{2\alpha}{1-\alpha}$.

Once we know the value of w^α , we introduce (55) into (52) and we find the second equation

$$\frac{e_s(e) \frac{1}{e} e_s(e)}{e_s(e)} = \tilde{A} \bar{\omega}; \quad (56)$$

where $e_s(e) > e_s(e)$; $\tilde{A} = \frac{J^\alpha(\tilde{A}; \gamma)}{1 - \gamma(J; \gamma) + J^\alpha(J; \gamma)}$ and $\frac{\partial \tilde{A}}{\partial \alpha} > 0$.

In order to obtain the third equation, we have to use the aggregate household budget constraint in period t . This budget constraint is:

$$C_t + A_{t+1} = (1 + r)A_t + w_t \bar{L} \quad (57)$$

where $C_t = C_t^t + C_t^{t-1}$.

We know that the wage and the level of consumption grow at the rate g . Hence, the wage at period $t + i$ is:

$$w_{t+i} = (1 + g)w_i \text{ or } w_{t+i} = (1 + g)^i w_t \quad (58)$$

The level of consumption is:

$$C_{t+i} = (1 + g)C_i \text{ or } C_{t+i} = (1 + g)^i C_t \quad (59)$$

Then we can write the budget constraint in the following way:

$$A_t + (1 + r)^{-1} \sum_{i=0}^{\infty} (1 + r)^{-i} w_{t+i} \bar{L} = (1 + r)^{-1} \sum_{i=0}^{\infty} (1 + r)^{-i} C_{t+i} \quad (60)$$

Since the total value of households assets must equal the total value of firms, we have

$$A_t = M_t Q = M_t \frac{q}{r} \quad (61)$$

Now, introducing (58), (59) and (61) in (60) we obtain

$$M_t \frac{q_t}{r} + (1 + r)^{-1} (\bar{L} w_t - C_t) \sum_{i=0}^{\infty} (1 + r)^{-i} (1 + g)^i = 0 \quad (62)$$

Because $\sum_{i=0}^{\infty} (1 + r)^{-i} (1 + g)^i = \frac{1+r}{r-g}$ and (43), we can write (62) in the following way:

$$\frac{M_t q_t}{r} + \frac{\bar{L} w_t}{\frac{1}{2}} = \frac{C_t}{\frac{1}{2}} \quad (63)$$

and introducing (45) in (63) we get

$$\frac{w_t J^{\frac{1}{1-\alpha}}(e)}{\frac{1}{2}} + \frac{\bar{L} w_t}{\frac{1}{2}} = \frac{C_t}{\frac{1}{2}} \quad (64)$$

We know from (57) and (60), respectively, that

$$C_t^t = \frac{w_t \bar{L}}{\mu} (1 - l) \quad (65)$$

$$\frac{C_{t+1}^t}{C_t^t} = \frac{1 + r}{1 + \frac{1}{2}} \quad (66)$$

Then,

$$C_{t+1}^t = \frac{\mu}{1 + \frac{1}{2}} \frac{1 + r}{\mu} \frac{w_{t+1} \bar{L} (1 - l)}{\mu} \quad (67)$$

Recalling that $C_t = C_t^t + C_{t+1}^t$ we get:

$$C_t = \frac{2 \bar{L} (1 - l)}{\mu} w_t \quad (68)$$

Finally, introducing (68) into (64), we find the third equation. This equation is:

$$\mu \frac{1}{2} + \mu \frac{1}{2} J^{\frac{1}{2}} \bar{L} = 2 \frac{1}{2} J^{\frac{1}{2}} \bar{L} (1 - l) \quad (69)$$

With the equations (49), (56) and (69) we can find the equilibrium values of l , e and n . Equations (56) and (69) give us the values of e and l respectively. These are:

$$e^* = \frac{J^{\frac{1}{2}} \bar{L}}{J^{\frac{1}{2}} \bar{L} + \mu \frac{1}{2}} \quad (70)$$

$$l^* = \frac{2 \frac{1}{2} J^{\frac{1}{2}} \bar{L}}{J^{\frac{1}{2}} \bar{L} + \mu \frac{1}{2}} \quad (71)$$

To find the value of n we substitute the values of l and e , defined by (70) and (71), in equation (49). Therefore,

$$n^* = \frac{2 \frac{1}{2} J^{\frac{1}{2}} \bar{L} + \mu \frac{1}{2} [(1 + \frac{1}{2}) \mu + 2 \frac{1}{2}]}{J^{\frac{1}{2}} \bar{L} (1 + \frac{1}{2}) + \mu \frac{1}{2} \bar{L}} \quad (72)$$

A diagrammatic representation of the equilibrium is given in Figure 1. The RR curve, equation (56), represents combinations of e and n for which the R&D sector is in equilibrium; the HH curve, equation (69), which represents combinations of l and e for which the households are in equilibrium; the LL curve, equation (49), represents the combinations of l , e and n for which the labour market is in equilibrium; and the GG curve, equation (41) represents iso-growth combinations of e and n .

The equilibrium will change when there are changes in the value of the parameters \bar{L} , μ , $\frac{1}{2}$, J , γ , and \bar{A} . From the expressions (56), (69) and (49) we can find the shifts in the curves RR, LL and GG, which show the effects of changes in the parameters on e , l and n and hence on g .

[Insert Figure 1]

The results for $\frac{1}{2}$, μ and \bar{L} are relatively trivial. An increase in the rate of time preference or taste for leisure, $\frac{1}{2}$ and μ respectively, reduces the growth rate ($\frac{\partial g}{\partial \frac{1}{2}} < 0$; $\frac{\partial g}{\partial \mu} < 0$) and employment. An increase in the number of households (i. e. a higher or larger economy) increases growth ($\frac{\partial g}{\partial \bar{L}} > 0$) and employment.

The results for \bar{A} , J and γ are more interesting. The effect of a higher \bar{A} is an increase in the fixed cost in R&D. This increase in the fixed cost leads to a negative expected profits and some firms will exit the R&D sector. Due to this exit of some firms, the rest of them will operate at a higher scale. However, the economy as a whole experiences less research activity. Therefore, the increase in \bar{A} has a negative effect on growth ($\frac{\partial g}{\partial \bar{A}} < 0$) and on employment. We can see the effect of an increase in \bar{A} in Figure 1. A higher \bar{A} shifts RR curve to the right and reaches $R'R'$. The other curves remain the same. Therefore, the result is higher e , lower n , lower l and lower g .

Finally, we will consider the effect of changes in the parameters which are a measure of labour variety (J and γ). Either greater J (the number of different types of labour) or lower γ (the degree of differentiation between types of labour or labour heterogeneity) has an ambiguous effect on growth ($\frac{\partial g}{\partial J} \gtrless 0$; $\frac{\partial g}{\partial \gamma} \gtrless 0$) and employment. This ambiguity is the consequence of a positive and a negative effect. The positive effect is the consequence of the increased efficiency of other production inputs which raises the demand for intermediate goods. This increase will attract firms towards the R&D sector. The negative effect comes from the increase of the fixed cost. This higher fixed cost impedes the entrance to new firms in the R&D sector. Therefore, the net effect depends on which effect dominates. Then, growth and employment

can either rise or fall. When τ decrease, we can see an additional negative effect. This additional effect is due to the increase in the degree of imperfect competition. The increase in the degree of imperfect competition reduces labour supply because trade unions take into account the greater trade-off between wages and employment.

4 POLICY IMPLICATIONS

From the model we can derive some policy implications. First we will consider subsidies in a close economy (financed by lump-sum taxes), and afterwards, we will open the economy in order to see the effect of trade liberalisation and labour market integration.

4.1 Subsidies Policy

We will consider two kinds of subsidy policies, subsidies to inputs and subsidies to outputs. In the subsidies to inputs we will differentiate subsidies to labour, subsidies to purchases of intermediate goods and subsidies to purchases of designs. In the subsidies to outputs we will refer to the effect of subsidies to the production of final goods, subsidies to the production of intermediate goods and subsidies to the production of designs.

4.1.1 Subsidies to Inputs

The effect of subsidies to labour on growth is ambiguous, but it has a positive effect on employment, independently of which labour is subsidised. If the labour of final goods or intermediate goods sector is subsidised, there will be a reallocation of labour. Workers will move from the R&D sector to the other two sectors, and this will cause a negative effect on growth. On the other hand, there is a positive effect as a result of the increase in the demand for intermediate goods, because the increase in demand will attract new firms to the R&D sector and growth will increase. With respect to employment, these subsidies have a positive effect because labour demand will increase, the wage will go up and labour supply will increase as well. All this means that each R&D firm operates at a smaller scale. If the subsidy is in the R&D sector the effects on growth is ambiguous, although opposite in direction to the effects comments above, and it will probably have a positive effect on employment.

In the other two kinds of input subsidies (that is, subsidies to purchases of intermediate goods or designs), the effect both on growth and employment is positive. In the case of subsidies to purchase of intermediate goods, the demand for labour and the wage will increase. These increases imply that a greater number of R&D firms will operate and there will be a positive response of the labour supply. If the purchase of designs is subsidised the effect on the number of R&D firms is similar to the effect of the previous case. However, now there is no change in the scale of operation of each firm.

In summary, we can say that the effect of subsidies to labour have stronger effects on employment and weaker effects on growth than the subsidies to purchases of intermediate goods or designs.

4.1.2 Subsidies to outputs

The effect of these kind of subsidies on growth and employment is positive because there will be a combination of an increase in the number of R&D firms and an increase in the demand for labour.

4.2 Trade Liberalisation

So far, we have considered a closed economy. Now, we will open the economy by removing the trade barriers. We will consider a world composed of K identical countries. Each country is an economy like the economy we showed in the previous section. We will assume that designs cannot be traded. Then, the production of intermediate goods is only possible in the country where the design is invented. We will also assume perfect patent protection which rule out the possibility of imitation and research redundancy. The type of trade we consider is the exchange of differentiated intermediate goods.

With trade liberalisation the number of intermediate goods available in each country is KM . Thus, the only difference between the model with trade liberalisation and the model of section 2 is that now the integrals are defined over the range $[0; KM]$. If we solve the model as before, we will see that the balanced equilibrium growth with trade liberalisation satisfies the same conditions as the equilibrium without free trade except that the constant growth adjustment wage is now $w^* = \frac{w}{KM}$, meaning that the term \bar{A} (the fixed cost) becomes $\frac{\bar{A}}{K}$. Therefore, the effect of trade liberalisation is like a reduction in the fixed cost.

As above, a reduction in the fixed cost causes an increase in the growth rate. So we can say, following Blackburn and Hung (1993), that the effect of trade liberalisation with the assumption of no knowledge spillover and no research redundancy is an expansion of the market for the designs as a consequence of trade liberalisation. The expansion of the market gives the possibility of entry of new firms in the R&D sector because there is positive expected profits.

We can also say that if the K economies are in a stagnation situation and they start free trade, then they can recover from stagnation. Moreover, if the number of countries engaging in free trade increases the effect is a bigger potential market and then, the expected profits for the R&D firms go up. The result is a greater number of R&D firms and then higher growth.

4.3 Labour Market integration

Like in the previous subsection, we have a world composed of K identical economies. But now, instead of introducing trade liberalisation we suppose that the K labour markets are integrated. To illustrate the effect of this in the simplest and starkest way possible, we make the assumption that the K economies trade only in differentiated types of labour, where the labour force of each economy is completely differentiated from the labour forces of other economies.

Given the above, the number of different types of labour (J) is replaced by KJ . Hence the effect of labour market integration on growth is like an increase in J . In consequence, because the increase in the number of different types of labour may produce a positive or negative effect on growth, as we saw in section 3, we can only say that the labour market integration has an ambiguous effect on growth, depending on which effect dominates. Therefore labour market integration may increase or decrease growth.

5 CONCLUSIONS

In this paper we sought to show how labour market imperfections might affect growth.

We developed a model in which research and development, asymmetric information and imperfect competition in the labour market combine to determine equilibrium rates of growth and employment. The motivation for the

paper was the lack of existing models which integrate labour market issues with growth, and the fact that labour market imperfections are characteristics of real world economies.

We found that, in general, an increase in labour variety and, in particular, an increase in the degree of labour market imperfections may either increase or decrease the rate of growth. The ambiguity was attributed to effects on factor efficiency information costs and labour supply which work in opposite directions. The role of information costs was motivated by an agency incentive problem which was solved through the formulation of contracts enforced by costly monitoring activity. The effect of the contract was to introduce a fixed cost into R&D which played a significant part in the subsequent analysis.

In particular, our model contained important scale effects which had important implications for policy. The existence of scale effects has been noted in previous literature and this paper provides one justification for them. Such effects draw attention to the importance of market size in determining growth.

We think that the scarce literature on this topic gives considerable scope for further research. We hope to pursue such research in the future.

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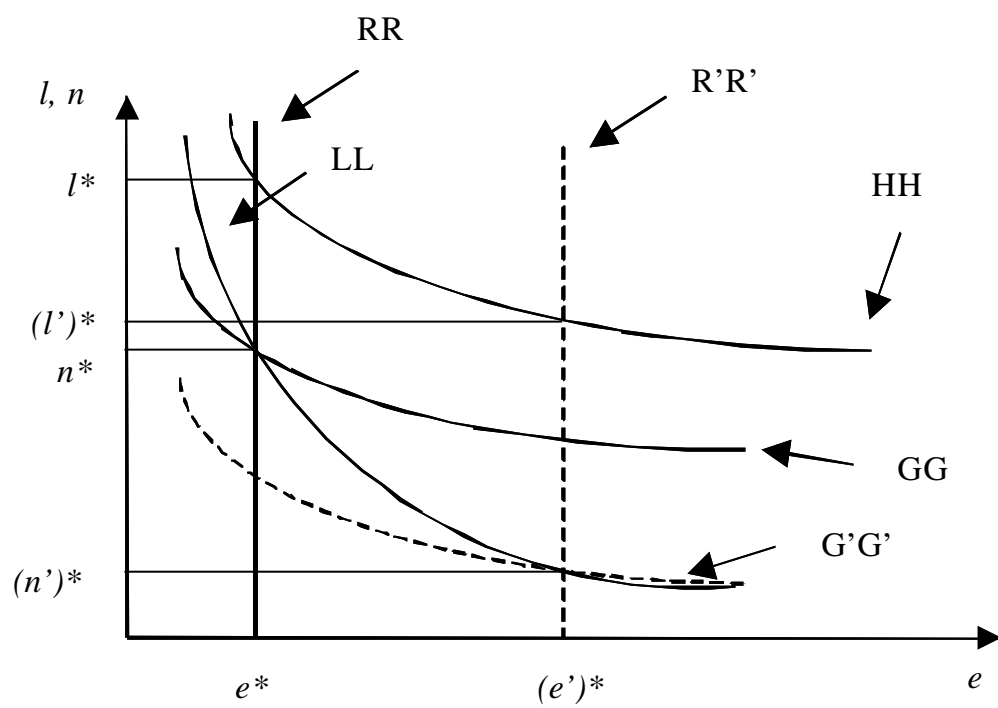


Figure 1: